Choose your auction:
Mechanism design for a bidder*

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Abstract

This study discusses the maximization of a bidder's utility in auctions by leveraging the information about a bidder's value to formulate the auction's rules. To make the analysis interesting, the research focuses on auction formats perceived as fair and unbiased, in line with common European Union or World Trade Organization procurement regulations. In the main model setup, we do not allow the auctioneer to pay the bidders and characterize a preferable auction format as a second-price auction which pools—i.e., does not distinguish between—certain sets of bids. The analysis is then extended to allow for transfers towards bidders. In this more permissible environment we demonstrate that a substantially positive net interim utility can be guaranteed to a bidder without running a deficit in equilibrium. The theory is applied to a model of favoritism, discussing whether forms of preferential treatment in auctions are preventable or detectable.

JEL: *D44, D73, D82.*

Keywords: *Auction design, Fair mechanism, Second-price auction, Lottery, Favoritism.*

1 Introduction

We consider the problem of utility maximization from one bidder's perspective, within the framework of auction design. Revenue and efficiency maximization are well-established and extensively researched topics within the theory of auctions and have been fundamental concerns since the early days of the mechanism design literature. We aim to augment this body of knowledge by assessing a bidder's objective in this context.

We study the maximization of a bidder's utility using the tools of mechanism design. Therefore, our model does not address any form of cheating that may arise after the auction design is announced, such as covert actions, unfair manipulation with bids, fictitious bidders, or other unjust actions. From our perspective, two elements render our analysis compelling. First, within the standard independent private values (IPV) setting, we assume that information about a particular bidder's value can be

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utilized in the auction design. This allows us to characterize a utility-maximizing auction for every possible value of that bidder. Second, we concentrate on auction formats that may commonly be perceived as fair and unbiased. The corresponding set of assumptions effectively prevents direct discrimination, such as allocating goods to a particular bidder, free of charge. Despite rendering the results of our analysis non-trivial, this restriction aligns naturally with practical considerations. For example, fairness and non-discrimination are essential prerequisites for procurement regulations. Institutions like the European Commission (EC) and the World Trade Organization (WTO) have established procurement guidelines to eliminate both positive and negative discrimination.[1]

Often, the auction literature requires in the context of non-discrimination that the employed mechanism has to be symmetric. In this setup, however, the symmetry of a mechanism may not be a binding constraint. Symmetric mechanisms may result in asymmetric equilibria, implementing an asymmetric social choice function in return (see, Chen and Knyazev, 2023). Based on this observation, Deb and Pai (2017) and Azrieli and Jain (2018) show that, for many auctions that are not symmetric, one can find a symmetric auction with a Bayes-Nash equilibrium with the same expected revenue and bidders' utilities. Thus, it is often possible to implement discriminatory outcomes in symmetric mechanisms. Hence, the symmetry of the mechanism may turn out to be insufficient to establish non-discrimination. To adhere to the motivation of our paper, we therefore require the mechanism to implement a symmetric social choice function. The primary contribution of this paper lies in demonstrating how the selection of an auction format can enhance the utility of a particular bidder while upholding the above restriction of non-discrimination.

To illustrate the paper's main idea, consider the following scenario: Suppose Anne partakes in an auction. She is faced with a choice between two formats: a standard second-price auction and a standard lottery. There is only one other bidder who has an independently drawn private value for the good. Denote Anne's valuation of the good by v^* and assume the opponent's value is distributed according to $F(v) = \sqrt{v}$. In the lottery case, Anne's expected utility is $v^*/2$, while in a second-price auction, her expected utility amounts to $2/3\,(v^*)^{3/2}$. Thus, Anne prefers a second-price auction over a lottery if her value v^* exceeds $9/16$, and conversely, she prefers a lottery over a second-price auction if her value v^* is less than $9/16$. Notably, both formats are non-discriminatory, ensuring that no bidder gains an unjust advantage. In this simple case, we can say that a *preferable auction plan* suggests selecting a lottery if Anne's value v^* is small and choosing a second-price auction if her value v^* is large.

The main result of this paper generalizes the above example. It characterizes the auction format preferable for a particular bidder with given value within the class of all Bayesian incentive-compatible (BIC) auctions that implement a symmetric social choice function. Our main finding demonstrates that, for any bidder's value v^*, a solution is a *second-price auction with pooling*. In this context, pooling refers to a mechanism that does not distinguish among bids within certain value ranges

[1] In particular, "*equal treatment, non-discrimination, mutual recognition, proportionality, and transparency*" are explicitly mentioned in a directive on public procurement by the European Commission (EC Directive on Public Procurement 26-Feb-2014). The WTO plurilateral Agreement on Government Procurement states that "*[e]ach party shall seek to avoid introducing or continuing discriminatory measures that distort open procurement*" (WTO Agreement on Government Procurement 30-Mar-2012).

and corresponds to a fair lottery when determining the auction winner. Furthermore, we present comparative static results to examine how variations in the bidder's value influence the selection of a specific auction mechanism. Notably, our analysis indicates that changes in the bidder's value have only a localized impact on the preferable auction format. The choice of a preferable auction mechanism demonstrates "continuity" concerning the bidder's value, exhibiting a smooth transition as the value varies.

Our study of this novel economic problem exhibits several compelling technical aspects. Differing from the conventional objectives of maximizing revenue or efficiency, our focus is maximizing utility for a specific bidder, resulting in an asymmetric optimization problem. Simultaneously, we seek to identify an auction that implements a symmetric social choice function and search for a symmetric solution to this inherently asymmetric problem. To address this, we utilize the robust majorization approach proposed by Kleiner et al. (2021). This method enables us to solve our mechanism design problem in a comprehensive setting, accommodating any given value v^* for which we tailor the auction, as well as any number of other bidders with independent and identically distributed values drawn from any distribution.

For our main characterization result, we do not allow the auctioneer to employ transfers to the bidders. We then elaborate upon additional possibilities to increase a specific bidder's utility by allowing the bidders to receive positive transfers from the auctioneer. In this part of the paper, we assume that an auction cannot run a deficit in equilibrium but may run a deficit off the equilibrium path, again excluding trivial cases in which auctions might incur infinite deficits. Our analysis demonstrates that even when we confine our attention to the class of strategy-proof mechanisms while permitting positive transfers to bidders, we can ensure a significant "net" interim utility that is at least equivalent to that bidder's value. To show this, we first demonstrate that collected revenues can be transferred to some bidder in equilibrium of any symmetric and strategy-proof auction. The key idea here is that every bidder submits their true value in equilibrium, and by leveraging knowledge of v^*, we can transfer auction revenues to a bidder whose bid aligns with the value v^*. A key challenge lies in designing the auction strategy-proof for the remaining bidders in this particular scenario. We demonstrate that with a suitable selection of transfer rules, it is feasible to achieve strategy-proofness.

Besides investigating a novel mechanism design problem, our analysis also offers practical insights into the issue of favoritism in auctions, analyzed from the perspective of mechanism design. Perceived in this way, we can interpret our model as describing a situation in which the designer chooses the auction format to maximize the utility of a favored bidder: Large companies may advocate for their preferred auction format through lobbying efforts. Procurement auctions involving international companies may also feature a local participant.[2] In a government spectrum auction where the government has partial or full ownership of a mobile company, the designer may prefer that

[2] Such "home biases" are well documented in European public procurement contracts. Herz and Varela-Irimia (2020) estimate that firms located in the home country of the tendering authority are about 900 times more likely to receive a contract.

particular company.[3] Our results contribute to a better understanding of the underlying mechanisms that enable such types of favoritism to persist despite efforts to establish fair and unbiased procurement processes. Besides favoritism, the application of our theoretical framework extends to various real-life scenarios where the auction format can be directly influenced by one of the bidders. For instance, Google actively participates as a bidder in its own advertisement auctions.[4]

In the context of favoritism, we address aspects of detection and prevention. While detecting favoritism resulting from the revenue transfer to a favored party can be relatively straightforward, this may become undetectable when favoritism arises from selecting a specific auction format from the class of non-discriminatory auctions. This negative result underscores the significance of favoritism prevention. Building upon the example developed above, if the designer is prohibited from utilizing a composite mechanism, consisting of both a lottery and second-price auction, the designer cannot use privileged information to favor some party and favoritism is unequivocally prevented. We establish that this rationale can be generalized: Including a single additional constraint that disallows stochastic allocation rules is sufficient to prevent any form of favoritism within our setting. Our results relating to favoritism remain robust also in cases of slight incompleteness in information regarding the preferred bidder's value.

Related literature

This paper is most directly related to the contributions by Deb and Pai (2017) and Azrieli and Jain (2018). The main difference to the present setup is our operationalization of the concept of fairness. While they deal with *procedural fairness*, requiring the employed mechanism to be symmetric, we require the *social choice function* implemented by some mechanism has to be symmetric itself. To avoid fostering asymmetric equilibria in the design of mechanisms, which may indirectly implement asymmetric allocations, we restrict our attention to mechanisms that ultimately implement symmetric social choice functions.

Our analysis explores BIC-direct auctions, drawing upon insights related to majorization as introduced by Kleiner et al. (2021). The concept of majorization also finds applications in other related contexts, such as the works of Ali et al. (2021), Bergemann et al. (2022), Gershkov et al. (2021a) and Gershkov et al. (2021b). Despite classifying an optimal auction within the class of Bayesian mechanisms, the identified second-price auction with pooling is considered strategy-proof.[5] This observation can be attributed to the findings of Manelli and Vincent (2010) and Gershkov et al. (2013), who establish the equivalence of Bayesian and strategy-proof implementations in terms of expected outcomes within the independent private values model.

[3] Extensive evidence supports the existence of substantial welfare losses caused by discriminatory practices in procurement. According to an estimate by Wensink and de Vet (2013), the costs of corruption in public procurement in eight EU countries range from €1.4 billion to €2.2 billion in 2010. According to OECD (2014), a substantial portion of foreign bribery cases involve the acquisition of public procurement contracts. Construction Sector Transparency Initiative (2021) indicates that mismanagement and corruption could result in losing 10-30% of investments in publicly funded construction projects.

[4] Google's participation in its own auctions is perceived as problematic; despite a favorable design of auctions that we have in mind, effects on prices are well documented (The Wall Street Journal, 19-Jan-2019).

[5] The set of strategy-proof mechanisms is a subset of Bayesian mechanisms.

Our applications to favoritism are closely connected to the extensive literature on collusion in auctions. Collusion among buyers (i.e., horizontal collusion) has been a prominent focus in auction theory, with path-breaking studies by Graham and Marshall (1987) and Mailath and Zemsky (1991) examining second-price auctions, and McAfee and McMillan (1992) investigating first-price auctions. Further research by Robinson (1985), Caillaud and Jéhiel (1998), Che and Kim (2006), Marshall and Marx (2007), and Che and Kim (2009) explores collusion possibilities among buyers across various auction formats. Collusion involving the seller and a buyer (i.e., vertical collusion) is studied in Compte et al. (2005), Menezes and Monteiro (2006), Burguet and Perry (2007), Arozamena and Weinschelbaum (2009), Burguet and Perry (2009), and Lengwiler and Wolfstetter (2010). However, these studies have primarily focused on specific auction formats and settings rather than perceiving it as a general mechanism design problem. Similarly, Arozamena et al. (2023) investigate favoritism issues arising from contract renegotiation while restricting their analysis to an unbiased auction format. To the best of our knowledge, applying our theory to favoritism aspects represents a first analysis of corruption and preferable treatment in auctions within the framework of general mechanism design.

Condorelli (2012) and Chakravarty and Kaplan (2013) have identified the social welfare maximizing mechanism with a benevolent designer in a setting where payments are wasted. They show that the optimal mechanism comprises contest and lottery regions depending on a distribution of values. When payments are wasted, we can view the problem of maximizing social welfare as equivalent to maximizing the utility of a specific bidder when their value and identity remain undisclosed. Therefore, in the context of our paper, their findings can be seen as a solution to a similar problem within a symmetric information setting, in which the information of a bidder's value can not be leveraged in the auction design.

A significant body of literature has explored the informed principal problem, including works by Myerson (1983), Maskin and Tirole (1990,9), Yilankaya (1999), Severinov (2008), and Mylovanov and Tröger (2012,1). In these models, the design of a mechanism can reflect the information that the designer possesses, allowing the revelation of this information to the agents in varying degrees. Our model involves the designer's knowledge of a bidder's value, which aligns with the informed principal literature. Since our objective is to find the best mechanism for a particular bidder in the IPV setting we want to avoid informational distortions. We therefore impose that bidders take the mechanism as exogenous in the main section.[6] However, we relax this assumption later in the section related to favoritism, but confine our attention to the class of strategy-proof mechanisms. Since each bidder has a dominant strategy in such mechanisms, they ignore the designer's revealed information about the favored bidder's presence, identity, and value.

The remainder of this paper is structured as follows. In the next section, we lay out the auction model. In Section 3, we present the main result of how a particular choice of a non-discriminatory auction can enhance the utility of a specific bidder. Section 4 extends the analysis by allowing for positive transfers towards bidders. In Section 5, we apply our theory to a model of favoritism

[6] Our results, however, do not depend on whether the bidder whose utility we want to maximize treats the mechanism as endogenous or exogenous.

in auctions. Issues of favoritism prevention are discussed in Subsection 5.1, matters related to the detection of favoritism in Subsection 5.2, and imperfect information settings are discussed in Subsection 5.3. Merely technical results are relegated to the Appendix.

2 The model

There is one indivisible good (object) that can be sold to a set $N = \{1, \ldots, n\}$ of bidders. Each bidder $i \in N$ is characterized by an independent private value, v_i, drawn from a continuously differentiable distribution F on $V = [0, \bar{v}]$ with positive density everywhere.[7] Denote by $\mathbf{v} = (v_1, \ldots, v_n)$ the vector of all values and by $\mathbf{v}_{-i} = (v_1, \ldots, v_{i-1}, v_{i+1}, \ldots, v_n)$ the vector of bidder i opponents' values. A *social choice function*, $\phi = (\mathbf{y}, \mathbf{t})$, is a collection of an *allocation rule*, $\mathbf{y} : [0, \bar{v}]^n \to [0, 1]^n$, with $\mathbf{y}(\mathbf{v}) = (y_1(\mathbf{v}), \ldots, y_n(\mathbf{v}))$, in which $y_i(\mathbf{v})$ is the probability that bidder i gets the good, and a *transfer rule*, $\mathbf{t} : [0, \bar{v}]^n \to \mathbb{R}^n$, with $\mathbf{t}(\mathbf{v}) = (t_1(\mathbf{v}), \ldots, t_n(\mathbf{v}))$, in which $t_i(\mathbf{v})$ specifies how much bidder i receives. Define $\mathbf{Y}$ as the set of all allocation rules and $\mathbf{T}$ as the set of all transfer rules. The ex-post utility of a bidder i is $U_i(\mathbf{v}) = v_i y_i(\mathbf{v}) + t_i(\mathbf{v})$. We define the interim allocation and the interim transfer for bidder i as

$$\bar{y}_i(v_i) := \mathbb{E}_{\mathbf{v}_{-i}} y_i(\mathbf{v}), \tag{1}$$
$$\bar{t}_i(v_i) := \mathbb{E}_{\mathbf{v}_{-i}} t_i(\mathbf{v}). \tag{2}$$

A vector of interim allocations, $\bar{\mathbf{y}} = (\bar{y}_1, \ldots, \bar{y}_n)$, is feasible if there exists an allocation rule, $\mathbf{y}$, that induces $\bar{\mathbf{y}}$ as its set of interim allocations, conditional on type. Then, the interim expected utility of bidder i,

$$U_i(v_i) := \mathbb{E}_{\mathbf{v}_{-i}} U_i(\mathbf{v}) = v_i \bar{y}_i(v_i) + \bar{t}_i(v_i),$$

represents the expectation of their ex-post utility taken with respect to a vector of other bidders' values, $\mathbf{v}_{-i}$. The reservation utility of all bidders is zero.

We apply the revelation principle and, without loss of generality, consider implementing social choice functions through the direct mechanisms where bidders are incentivized to report their values in Bayes-Nash equilibrium truthfully. For simplicity and with a slight abuse of notation, we use $(\mathbf{y}, \mathbf{t})$ to denote the direct mechanism that implements a social choice function ϕ. The well-known characterization of Myerson (1981) provides the necessary and sufficient conditions for the implementability of a social choice function. A social choice function ϕ is implementable, or in other words, a corresponding direct mechanism is BIC, if and only if, for each bidder i and for all $v_i \in [0, 1]$, it holds that

$$\bar{y}_i(v_i) \text{ is non-decreasing in } v_i, \text{(monotonicity)} \tag{3}$$
$$v_i \bar{y}_i(v_i) + \bar{t}_i(v_i) = h_i + \int_0^{v_i} \bar{y}_i(q) \mathrm{d}q, \text{ (envelope)} \tag{4}$$

in which h_i is a constant that does not depend on bidder i's value.

Our objective is to determine the auction format that maximizes the utility of a particular bidder, and without loss of generality, we assume that this is bidder 1. We denote by $v^* = v_1$ the realization

[7] With minor technical modifications, our analysis extends naturally to the case of unbounded distributions.

of bidder 1's private value. Our goal is to select the auction format based on this specific value realization to maximize bidder 1's expected utility, represented as $U_1(v^*)$. We say that $(\mathbf{y}^*, \mathbf{t}^*)$ is a *preferable auction* for v^* if it maximizes the interim utility of bidder 1, $U_1(v^*)$, among all direct mechanisms. Without any further constraints, the good could be simply allocated to bidder 1 free of charge. To prevent such direct discrimination, we impose the following symmetry constraints.

We restrict our analysis to mechanisms that implement symmetric social choice functions. Let $\pi : \{1, \ldots, n\} \rightarrow \{1, \ldots, n\}$ be a permutation. Thus, $\pi(i) = j$ means that the element in j-th position moves to i-th position when permutation π is applied. Denote by Θ^n the set of all permutations of n elements and denote a permuted vector of values by $\pi\mathbf{v} := (v_{\pi(1)}, \ldots, v_{\pi(i)}, \ldots, v_{\pi(n)})$.

Definition 1 (Symmetry[8])**.** *A social choice function, ϕ, is symmetric if, for any bidder $i \in N$, for any permutation, $\pi \in \Theta^n$, and for any vector of values, $\mathbf{v} \in [0, \bar{v}]^n$*

$$y_i(\pi\mathbf{v}) = y_{\pi(i)}(\mathbf{v}), \tag{5}$$
$$t_i(\pi\mathbf{v}) = t_{\pi(i)}(\mathbf{v}). \tag{6}$$

We say that the auction $(\mathbf{y}, \mathbf{t})$ is symmetric if the social choice function $\phi(\mathbf{y}, \mathbf{t})$ it implements is symmetric. Notably, symmetry holds when the bidders' identities have no bearing on the outcome, specifically when any permutation of bids among the bidders results in a symmetrical alteration of $(\mathbf{y}, \mathbf{t})$. This definition implies that if bidder i, following a permutation π, places a bid that agent $\pi(i)$ made prior to the permutation, they should have an equal probability of winning the auction and receiving the same transfer as agent $\pi(i)$ did before the permutation. Notice that symmetry implies that $h_i := h$ for any $i \in N$ in (4).

In the absence of any further transfer constraints, it is feasible to implement a social choice function that provides all bidders with infinitely large positive transfers, leading to an unbounded auction deficit. In the main part of the paper we therefore restrict attention to the case where positive transfers to the bidders are not allowed, thereby placing the following constraint:[9]

$$\mathbf{t}(\mathbf{v}) \leq \mathbf{0}, \quad \forall \mathbf{v} \in [0, \bar{v}]^n. \tag{7}$$

We say that the auction $(\mathbf{y}, \mathbf{t})$ exhibits positive transfers when the social choice function it implements exhibits positive transfers. With these restrictions in hand, we are ready to formulate our main objective, i.e., we want to specify the *preferable auction*, $(\mathbf{y}^*, \mathbf{t}^*)$, that solves

$$\max_{\mathbf{y}, \mathbf{t} \in (\mathbf{Y}, \mathbf{T})} \quad U_1(v^*) \tag{8}$$

s. t. : (1), (2) (relationship between interim and ex-post rules)

(3) (monotonicity of interim allocation)

(4) (envelope condition)

(5), (6) (symmetry of the rules)

(7) (non-positive transfers)

[8] Note that *symmetry* is sometimes referred as *anonymity* in the literature.

[9] Note that if we substitute the ex-post constraint on transfers with an ex-ante constraint, our analysis yields the same results throughout our paper.

Finally, for varying v^*, we define an *auction plan* $M : V \to \mathbf{Y} \times \mathbf{T}$ that maps every possible value realization of v^* to an auction. We say that the auction plan M^* is a *preferable plan* of auctions if $M^*(v^*) = (\mathbf{y}^*, \mathbf{t}^*)$ for every v^*, i.e., this plan chooses the preferable auction for every realization of bidder 1's value. The *preferable plan* of auctions is the key object that we study in this paper.

3 Characterization of the preferable auction

In this section, we provide our main result and characterize the a solution to problem (8). For that, we need some additional notations. For any realization $v^* \in V$, denote by $G_{v^*} : V \to \mathbb{R}_+$,[10]

$$G_{v^*}(z) = \begin{cases} \mathrm{conc}_{v^*} \langle F(z) \rangle & \text{for } z \leq v^* \\ 1 + \left(\lim\limits_{q \nearrow v^*} \dfrac{\mathrm{d}G_{v^*}(q)}{\mathrm{d}z} \right)(z - v^*) & \text{for } z > v^* \end{cases}$$

in which $\mathrm{conc}_{v^*} \langle F(z) \rangle$ is the smallest function that is concave, weakly greater than $F(v)$, and takes a value of 1 at $z = v^*$. Furthermore, denote by

$$g_{v^*}(z) := \frac{\mathrm{d}G_{v^*}(z)}{\mathrm{d}z}, \tag{9}$$

noticing that $G_{v^*}(z)$ is differentiable on V by construction. Now, we are able to formulate our main result and characterize a preferable auction.

Proposition 1 (Preferable auction)**.**
For each value realization v^*, the symmetric preferable auction plan, $M^*(v^*)$, allocates the object to a bidder with the smallest $g_{v^*}(v_i)$. Ties are broken with a simple lottery to determine a winner. Transfers are computed by (4) *with $h = 0$.*

Intuitively, the concavity of G_{v^*} implies that g_{v^*} decreases weakly, thereby establishing a direct relationship between a lower value of $g_{v^*}(v_i)$ and a higher value of v_i. The auction outlined in Proposition 1 possesses a clear economic description and is easy to implement. We refer to it as a *second-price auction with pooling*. Conceptually, this is a standard second-price auction with the modification that the value domain has some intervals where the mechanism does not distinguish between the bids within any interval. Consequently, the optimal mechanism for any distribution F and value v^* can be characterized by the number and positioning of these pooling regions.

Example 1:
An example of the optimal auction is illustrated in Figure 1. In this particular case, there are two pooling regions: the bottom region, $[0, a)$, and the top region, $[b, \bar{v})$. Bidder 1's value, v^*, always falls within the top pooling region, and the auction's outcome depends on the opponents' values.
If opponents have values larger than v^* or slightly below that, the winner is determined by a lottery among values in $[b, \bar{v})$. This allocation rule ensures that, in any case bidder 1 has a strictly positive probability of winning the good. The winner has to pay the lower boundary of the top pooling region, b, in Figure 1. This payment serves two purposes: on the one hand, it limits the participation of

[10] $\lim_{q \nearrow v^*}$ is the limit from below at v^*. We use it to define $G_{v^*}(z)$ for values beyond $[0, 1]$.

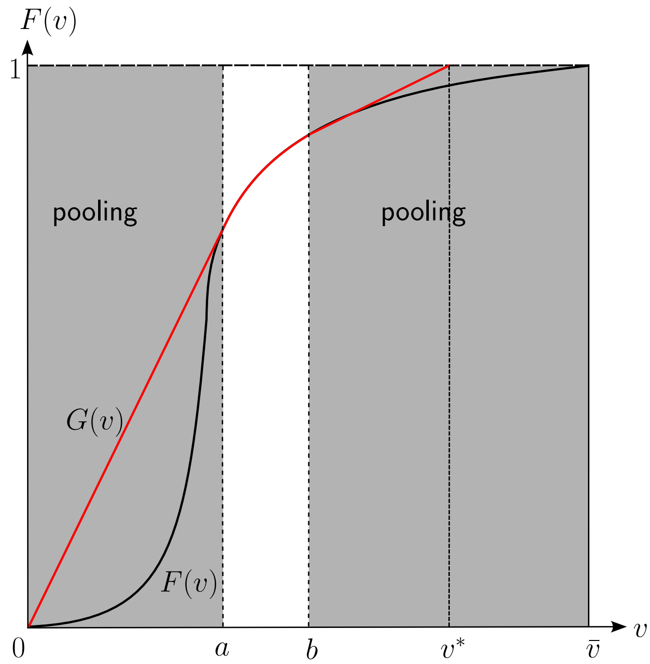

Figure 1: Second-price auction with pooling

bidders with small values in the lottery; on the other hand, it ensures that bidder 1 still receives a positive utility of $v^* - b$ in case of winning.[11] If the highest opponents' value, v, is in $[a, b)$, the mechanism behaves like a second-price auction, where bidder 1 is the winner and pays the price equal to v. The fact that $v < b < v^*$ ensures that bidder 1 pays a sufficiently low price v. Finally, the existence of the lower pooling region, $[0, a)$, allows for a decrease in bidder 1's expected payment. Since $v^* > a$, bidder 1's value is above this pooling region. Bidder 1 gets the good and pays $a(n-1)/n$, yielding a lower expected payment compared to a standard second-price auction, i.e.,

$$a(n-1)/n < \mathbb{E}[v_{(1)}^{(n-1)} | v_{(1)}^{(n-1)} < a].$$

This can be seen in Figure 1, observing that F is strictly below its envelope in the interval $[0, a)$. ◁
The example represented in Figure 1 illustrates two reasons why pooling arises generally in the preferable auction. The first is that it sometimes allows bidder 1 to get the object when the opponents' values are higher. The second reason pooling may be optimal is that it reduces the expected payment made by bidder 1 when their value is substantially higher than that of their competitors. Below, we provide the proof of Proposition 1, based on the majorization approach introduced by Kleiner et al. (2021).

Proof of Proposition 1.
For each v^*, the preferable auction plan, $M^*(v^*)$, should maximize bidder 1's utility

$$U_1(v^*) = v^* \bar{y}_1(v^*) + \bar{t}_1(v^*).$$

[11] The fact that the winner has to pay the lower bound of the top pooling region is the reason why this lower bound is not equal to v^* but is strictly below.

Using (4), and Definition 1, $U_1(v^*)$ can be rewritten as

$$U_1(v^*) = h + \int_0^{v^*} \bar{y}_1(v)\mathrm{d}v.$$

Inequality (7) implies $\mathbf{t}(\mathbf{v}) \leq \mathbf{0}$. Using (4),

$$0 \geq \bar{t}_1(0) = h.$$

Thus, by maximizing utility of bidder 1 in a preferable auction M^*, we have $h = 0$ giving in return

$$U_1(v^*) = \int_0^{v^*} \bar{y}_1(v)\mathrm{d}v.$$

For any value v define the quantile $s = F(v)$, and

$$\psi(s) = \bar{y}_1(F^{-1}(s)), \tag{10}$$

as the interim quantile allocation induced by the interim allocation, $\bar{y}_1(v)$. It is important to note that an allocation, $\bar{y}_1(v)$, is part of a BIC mechanism if and only if the induced interim quantile allocation, $\psi(s)$, is non-decreasing (Gershkov and Moldovanu, 2010; Kleiner et al., 2021). Then, as

$$\frac{\mathrm{d}s}{\mathrm{d}v} = \frac{1}{f(F^{-1}(s))},$$

integration by substitution gives

$$\int_0^{v^*} \bar{y}_1(v)\mathrm{d}v = \int_0^{F(v^*)} \frac{\psi(s)}{f(F^{-1}(s))}\mathrm{d}s. \tag{11}$$

Define ψ^e as the interim quantile allocation for bidder 1 induced by the efficient allocation y^e, in which the bidder with the highest value gets the good, and ties are broken by fair randomization. To solve our relaxed problem, we use a modern approach of majorization introduced by Kleiner et al. (2021). For two non-decreasing functions $\rho, \sigma \in L^1$, we say that ρ weakly majorizes σ, denoted by $\sigma \prec_w \rho$, if the following condition holds

$$\int_x^1 \sigma(s)\mathrm{d}s \leq \int_x^1 \rho(s)\mathrm{d}s, \quad \text{for all } x \in [0,1].$$

Next, for any $\rho \in L^1$ we define the set of all non-decreasing and non-negative functions that are weakly majorized by ρ

$$\mathrm{MPS}_w(\rho) = \{\sigma \in L^1 | \sigma \text{ is non-decreasing and non-negative such that } \sigma \prec_w \rho\}.$$

By Theorem 3 from Kleiner et al. (2021), any symmetric and monotone interim quantile allocation, $\psi(s)$, is feasible if and only if it is weakly majorized by ψ^e. Thus, $\psi \in \mathrm{MPS}_w(\psi^e)$. Then, the objective of the designer can be written as

$$\max_{\psi\, \in \mathrm{MPS}_w(\psi^e)} \int_0^{F(v^*)} \frac{\psi(s)}{f(F^{-1}(s))}\mathrm{d}s = \max_{\psi \in \mathrm{MPS}_w(\psi^e)} \int_0^1 \frac{\psi(s)}{f(F^{-1}(s))}\mathbf{1}_{s \leq F(v^*)}\mathrm{d}s, \tag{12}$$

in which $\mathbf{1}_{s\leq F(v^*)}$ is the indicator function such that

$$\mathbf{1}_{s\ \leq F(v^*)} = \begin{cases} 1 & \text{if } s \leq F(v^*), \\ 0 & \text{otherwise.} \end{cases}$$

We say that $x \in A$, in which A is convex, is *extreme* if $x = \alpha y + (1-\alpha)z$ for $z, y \in A$ and $\alpha \in [0,1]$ implies $x = y$ or $x = z$. Denote by ψ^* a solution to (12). Now, following Kleiner et al. (2021) we want to apply Bauer's Maximum Principle: a convex, upper semi-continuous functional on a non-empty, compact, and convex set A attains its maximum at an extreme point of A. Since ψ^e is non-decreasing, Proposition 1 of Kleiner et al. (2021) implies that $\text{MPS}_w(\psi^e)$ is convex and compact and the set of extreme points of $\text{MPS}_w(\psi^e)$ is non-empty. Notice also that the problem (12) is linear in ψ. Then, Bauer's Maximum Principle implies that ψ^* is the extreme point of $\text{MPS}_w(\psi^e)$. Then, Theorem 1 of Kleiner et al. (2021) implies that there exists a collection of intervals, $[\underline{x}_j, \overline{x}_j)_{j=1}^J$, such that for almost every $s \in [\underline{x}_j, \overline{x}_j)$

$$\psi^*(s) = \begin{cases} \psi^e(s) & \text{if } s \notin \bigcup_j [\underline{x}_j, \overline{x}_j), \\ \frac{\int_{\underline{x}_j}^{\overline{x}_j} \psi^e(s)\mathrm{d}s}{\overline{x}_j - \underline{x}_j} & \text{if } s \in [\underline{x}_j, \overline{x}_j). \end{cases}$$

Similarly to equation (8) in Kleiner et al. (2021), we define

$$\Psi_{v^*}(s) = \int_0^s \frac{1}{f(F^{-1}(q))} \mathbf{1}_{q\leq F(v^*)} \mathrm{d}q.$$

Notice that for $s \leq F(v^*)$ we have

$$\int_0^s \frac{1}{f(F^{-1}(q))} \mathbf{1}_{q\ \leq F(v^*)} \mathrm{d}q = \int_0^s \frac{1}{f(F^{-1}(q))} \mathrm{d}q = F^{-1}(s)$$

and for $s > F(v^*)$ we have

$$\int_0^s \frac{1}{f(F^{-1}(q))} \mathbf{1}_{q\ \leq F(v^*)} \mathrm{d}q = \int_0^{F(v^*)} \frac{1}{f(F^{-1}(q))} \mathrm{d}q = F^{-1}(F(v^*)) = v^*,$$

thus,

$$\Psi_{v^*}(s) = \begin{cases} F^{-1}(s) & \text{if } s \leq F(v^*), \\ v^* & \text{if } s > F(v^*). \end{cases}$$

By $\text{conv}\langle\Psi_{v^*}\rangle$ denote the convex hull of Ψ_{v^*}, that is, the largest convex function that lies below Ψ_{v^*}. Since ψ^e is strictly increasing because of smooth F and ψ^* is an extreme point of $\text{MPS}_w(\psi^e)$, Proposition 2 of Kleiner et al. (2021) implies that ψ^* is a solution to (12) if and only if $\text{conv}\langle\Psi_{v^*}\rangle$ is affine on $[\underline{x}_j, \overline{x}_j)$ for each $j \in J$ and $\text{conv}\,\langle\Psi_{v^*}\rangle = \Psi_{v^*}$ otherwise. Finally, notice that $(\text{conv}\langle\Psi_{v^*}\rangle)^{-1} = \text{conc}_{v^*}\langle F\rangle$. Thus, ψ^* is implemented by the proposed auction format. ☐

The next set of results summarizes some properties of the preferable auction.

Proposition 2 (Properties of the preferable auction).
Consider the preferable auction plan $M^*(v^*)$ from Proposition 1.

1. *For any bidder 1's value realization $v^* < \bar{v}$, there exists a cutoff $\hat{v} < v^*$, such that the preferable auction pools all bidders with values above $\hat{v}$.*

2. *The cutoff $\hat{v}(v^*)$ is a monotone increasing function of v^*.*

3. *The only difference between auctions chosen by the optimal plan M^* for different values of v^* is the size of a pooling region determined by the cutoff function $\hat{v}(v^*)$. The lower v^* is, the larger this pooling region is.*

4. *If $f(0) < \infty$, there exists a cutoff v_l such that $M^*(v^*)$ is a simple lottery for any $v^* \leq v_l$.*

First, for any given value v^*, the optimal auction plan, $M^*(v^*)$, always incorporates pooling between bidder 1 and all bidders with higher values. Additionally, it includes some bidders with lower values that exceed the cutoff threshold, $\hat{v}(v^*)$. Then, if v^* gets smaller, bidder 1 wants to increase pooling in the region of high values and keep the same allocation rule for low realizations of values. Hence, the change of v^* has only a local effect on the auction design. Moreover, when v^* is sufficiently small, the entire value range is pooled. Consequently, a simple lottery mechanism becomes the optimal choice in such cases.
The characterization of the optimal mechanism is simpler for distributions F that are convex, linear or concave.

Corollary 1 (Simple shape distributions)**.**
Consider the preferable auction plan, $M^*(v^*)$, from Proposition 1.

1. *If F is convex or linear, $M^*(v^*)$ is a standard lottery for any bidder 1's value realization $v^* \in V$.*

2. *If F is concave, $M^*(v^*)$ is a second-price auction with pooling at the top determined by $\hat{v}(v^*)$.*

In cases where the distribution F is convex, the optimal auction aligns with the optimal auction for small values of v^*. In this scenario, bidder 1 finds it excessively costly to compete with their opponents and would instead prefer a lottery, even if possessing the highest possible value. If, by contrast, F is concave, bidder 1 preferres a mechanism where a lottery is employed only when their opponents have higher values than v^* or values that are sufficiently close.

Remark 1. *Note that the preferable auction plan characterized in Propositions 1 and 2 is a strategy-proof direct auction defined as follows*

1. *An auction is strategy-proof if, for any bidder, truthful reporting provides higher interim utility than any value misrepresentation, namely, for all $i, v_i, \tilde{v}_i$ and $\mathbf{v}_{-i}$*

$$v_i y_i(v_i, \mathbf{v}_{-i}) + t_i(v_i, \mathbf{v}_{-i}) \geq v_i y_i(\tilde{v}_i, \mathbf{v}_{-i}) + t_i(\tilde{v}_i, \mathbf{v}_{-i}).$$

2. *An auction plan M is strategy-proof if $M(v^*)$ is strategy-proof for any $v^* \in V$.*

This observation can be directly derived from the fact that the solution presented in Proposition 1 corresponds to a mixture of a second-price auction and a lottery, both well-known strategy-proof mechanisms. This is, however, not surprising and aligns with the findings of Gershkov et al. (2013), who establish the equivalence between symmetric direct BIC mechanisms and symmetric direct strategy-proof mechanisms.[12]

4 Auctions with positive transfers to bidders

In this section, we want to further elaborate upon possibilities to maximize the payoff of a bidder, upholding the restrictions on symmetric social choice functions (5) and (6) but relaxing the restrictions on transfers (7). In practice, however, it may be hard to justify any format that results in a negative revenue for the seller. Therefore, we still impose a weak restriction on transfers

$$\sum_{i=1}^{n} t_i(v^*, \mathbf{v}_{-1}) \leq 0 \text{ for any } \mathbf{v}_{-1}. \tag{13}$$

Thus, rather than (7), we limit attention to auction formats that do not run an *ex-post deficit on the equilibrium path*. In order to define a benchmark for the utility of bidder 1 in such auction formats, we define

Definition 2 (Perfect auction)**.** *An auction plan M is perfect for bidder 1 if, for any value realization v^*, the auction plan $M(v^*)$ provides $U_1(v^*) \geq v^*$ almost surely.*[13]

Thus, a perfect auction can almost surely provide bidder 1 with an interim utility greater than or equal to their value of the good. A straightforward instance of a *perfect* auction, without symmetry constraints (5) and (6), allocates the good to bidder 1 regardless of the bids made. Another simple instance of a *perfect* auction transfers a guaranteed payment of $t_1 \geq v^*$ to bidder 1. Naturally, such auction rules are discriminatory. However, as we shall demonstrate below, a *perfect auction* is possible within the class of symmetric and strategy-proof auctions. This claim depends on the auction creating a sufficiently high ex-post revenue.

Proposition 3 (Revenue transfers)**.**
For any direct symmetric and strategy-proof auction $(\mathbf{y}, \mathbf{t})$ that generates the ex-post equilibrium revenue $R(v^*, \mathbf{v}_{-1}) = -\sum_{i=1}^{n} t_i(v^*, \mathbf{v}_{-1})$, given bidder 1's value realization v^*, there exists another transfer rule $\mathbf{t}'$ such that the direct auction $(\mathbf{y}, \mathbf{t}')$ is

(i) symmetric and strategy-proof, and

(ii) almost surely implements the same equilibrium transfers for all bidders except for bidder 1.[14]

[12] Notice that the identified preferred auction plan is non-unique among the class of BIC mechanisms. There are some BIC mechanisms, which are also optimal but not strategy-proof while implementing the same interim allocation and transfer rules, e.g., a first-price auction with pooling or an all-pay auction with pooling.

[13] Almost surely, means here that the event happens with probability one, i.e., $\mathbb{P}\{U_1(v^*) \geq v^*\} = 1$.

[14] More precisely this means that $t'_j(v^*, \mathbf{v}_{-1}) = t_j(v^*, \mathbf{v}_{-1})$ for any $j \neq 1$, and bidder 1's equilibrium transfer is such that $t'_1(v^*, \mathbf{v}_{-1}) = t_1(v^*, \mathbf{v}_{-1}) + R(v^*, \mathbf{v}_{-1})$ almost surely.

In other words, an auction can transfer all collected revenue to bidder 1 in equilibrium almost surely, even in a symmetric and strategy-proof auction. Almost surely here implies that the statement is true for all realizations of values, except for those where one or more bidders' values coincide with bidder 1's value v^*. However, since the distributions of values are absolutely continuous and the number of bidders is finite, the probability of such an event is equal to zero. The reasoning behind this finding is that auction regulations can be manipulated, considering that bidder 1 places a bid of v^*. Consequently, the auction is designed to make it advantageous to bid truthfully for any given value, resulting in the transfer of all revenues to the bidder who submitted a bid of v^*, namely bidder 1. Since the probability that more than one bidder has a value v^* is zero, these instances do not affect bidder 1's benefit. To ensure strategy-proofness, substantially high payments should be guaranteed to bidders when no one bids v^*. Generally, if there is no bidder reporting v^*, the auction would not be budget-balanced and would run a huge deficit. However, since bidder 1 reports v^*, the auction is always balanced on the equilibrium path.
Now, consider a standard second-price auction. It is symmetric and strategy-proof. Hence, Proposition 3 implies the following result.

Corollary 2 (Transferring revenue in the second-price auction).
There exists a symmetric and strategy-proof auction $(\mathbf{y}, \mathbf{t}')$ that implements the same allocation rule for all bidders and the same transfers in equilibrium for all bidders except for bidder 1 as the second-price auction. Instead, under transfers $\mathbf{t}'$, bidder 1 receives all collected revenue and almost surely has an ex-post utility greater than the value v^* in equilibrium.

It is possible to transfer the revenue collected in a second-price auction to bidder 1 for almost all opponents' bids. In such an auction, bidder 1 wins if and only if they have the highest value and pays nothing in this case almost surely. If the value of bidder 1 is not the highest, then the bidder with the highest value obtains the good, and bidder 1 receives a transfer equal to the second-highest value. Thus, in all cases when bidder 1's value is not the highest or the second-highest one, the utility obtained by bidder 1 is equal to the second-highest value and strictly exceeds v^*. Thus, the following statement is true.

Corollary 3 (Symmetric and strategy-proof perfect auction).
A perfect auction exists within the class of symmetric and strategy-proof auctions.

Remark 2. *The auction proposed in Corollary 3 is efficient, i.e., the good is always allocated to the bidder with the highest value. Therefore, we can extend the above corollary to the class of efficient auctions.*

5 Favoritism in auctions

In this section, we apply our preceding results to a model of favoritism. We define *favoritism* as a scenario in which the auction designer aims to maximize the utility of a specific, favored bidder due to various reasons. As discussed in the introduction, the designer may have received a bribe from the favored bidder to select the auction format that best suits that bidder's interests. Considering this notion of favoritism, the designer seeks to maximize the utility of the favored bidder. Consequently, the designer's and the favored bidder's incentives are perfectly aligned, and the favored bidder would willingly disclose information about their value to the designer.

As argued previously, even if direct discrimination within the auction can be ruled out, the designer can still adopt a mechanism conditional on the value of the favored bidder. We demonstrate below that such forms of favoritism can be prevented by imposing two additional constraints on the auction design in addition to symmetry. We then discuss if forms of preferential treatment are detectable by an external regulatory authority. In the subsequent chapters we confine our attention to the class of strategy-proof mechanisms to circumvent issues associated with informed principle problems.[15].

5.1 Prevention of favoritism

We start our analysis of favoritism prevention by defining a class of auction rules that excludes randomized allocations in the absence of ties.

Definition 3 (Deterministic auctions)**.** *An auction, $(\mathbf{y}, \mathbf{t})$, is deterministic if, for any bidder i such that $0 < y_i(\mathbf{b}) < 1$, there exists another bidder, $j \neq i$, such that $b_j = b_i$.*

This definition states that in situations where a bidder's probability of obtaining a good is not absolutely certain (0 or 1), at least one other bidder has made an identical bid. An illustration of a deterministic auction is the conventional second-price auction. In contrast, a second-price auction incorporating pooling, as outlined in Proposition 1, does not adhere to determinism because it uses randomization to determine the winner, even when all bidders report different bids. To proceed further, we first introduce the following class of auctions.

Definition 4 (Second-price auction with a flexible reserve price)**.** *An auction is a second-price auction with a flexible reserve price if*

$$y_i(\mathbf{v}) = \begin{cases} 1 & \textit{if } v_i > \max_{j \neq i}(v_j, r(\mathbf{v}_{-i})) \\ 0 & \textit{otherwise} \end{cases}, \textit{ and}$$

$$t_i(\mathbf{v}) = \begin{cases} -\max_{j \neq i}(v_j, r(\mathbf{v}_{-i}))\} & \textit{if } v_i > \max_{j \neq i}(v_j, r(\mathbf{v}_{-i})) \\ 0 & \textit{otherwise} \end{cases},$$

in which $r : \mathbb{R}^{n-1} \to \mathbb{R}_+$ is a component-wise symmetric function.[16]

[15] When the mechanism designer possesses private information, the design of the mechanism may reveal information about the favored bidder's value. This, in turn, could affect the bidding strategies of other bidders. Since we restrict our attention to strategy-proof mechanisms, however, truthful bidding is always optimal for any bidder. This precludes players from strategically exhausting this information. For details, we refer to the succinct Myerson (1983) and the more recent Mylovanov and Tröger (2012,1)

[16] In the zero probability case of two or more equal highest bids, symmetry implies that only a symmetric lottery can determine the winner who obtains the object and pays the bid if it is greater than a generalized reserve price. Note also that the reserve price is the same for all agents with the highest bids due to symmetry.

The difference between a second-price auction with a flexible reserve price and a standard second-price auction is that the reserve price is not the same for different bidders. For the winning player, it may depend on bids made by their opponents. If a flexible reserve price is a constant function, i.e., $r(\mathbf{v}_{-i}) = c$, $c \in \mathbb{R}$, $\forall \mathbf{v}_{-i} \in \mathbb{R}^{n-1}$, we obtain a second-price auction with a standard reserve price. In the special case of a zero reserve price, we get a standard second-price auction. The following result, due to Chen and Knyazev (2023), characterizes the set of all deterministic symmetric and strategy-proof direct auctions.

Proposition 4 (Chen and Knyazev (2023))**.**
A direct auction is deterministic, symmetric, strategy-proof, and satisfies $U_i(0, \mathbf{v}_{-i}) = 0$, if and only if it is a second-price auction with a flexible reserve price.

The following result shows there is no scope for the designer to choose the auction if they can use only deterministic, anonymous, and strategy-proof auctions.

Proposition 5 (No favoritism)**.**
The deterministic, symmetric, and strategy-proof preferable auction is a second-price auction with a reserve price of zero.

Given the constraints on positive transfers established in Proposition 4, the designer's options are limited to selecting an auction from the set of second-price auctions with a flexible reserve price. Proposition 5 demonstrates that setting the reserve price to zero most benefits the favored bidder. Therefore, the optimal choice for the designer is to consistently opt for a second-price auction, independent of the favored bidder's value and the value distribution. It is worth pointing out that even though the designer's freedom to choose an auction format is restricted, Myerson's revenue-maximizing auction can still be employed if the bidders are symmetric. In fact, the revenue-maximizing auction corresponds to a second-price auction with a standard reserve price that falls within the set of second-price auctions with flexible reserve prices in this environment. However, if bidders are heterogeneous, Myerson's revenue-maximizing auction is not symmetric and thus unavailable to a designer. In such a case, Chen and Knyazev (2023) show that a second-prize auction with a flexible reserve price is revenue-maximizing if bidders are heterogeneous and the designer has to use symmetric direct auctions.

5.2 Detection of favoritism

Our findings suggest that a designer can employ non-discriminatory auctions, ideally the optimal mechanism outlined in Proposition 1, to advance the interests of a favored bidder without engaging in any direct discriminatory practices during the auction. An essential question seems then to revolve around the identification of favoritism in such contexts. Intuitively, detecting favoritism becomes unfeasible when the auction is conducted only once; it is impossible to ascertain whether a simple lottery is implemented due to fairness considerations or favoritism.

In order to develop this notion, we introduce a regulatory entity that seeks to determine whether an auction exhibits favoritism. We assume that this regulator is cognizant of the possibility of favoritism in an auction but lacks knowledge regarding the identity of the favored bidders. Furthermore, we assume that the regulator does not know the market and is thus unaware of the distribution of bidders' values, F, yet possesses the ability to observe the auction format and the bids submitted by all bidders.[17] We assume that the regulator constructs a belief, $\Phi : \mathbf{Y} \times \mathbf{T} \times V^n \to \Delta(N)$, regarding the identity of a favored bidder based on the observed bids, auction rules, and outcomes. Such a belief maps the given auction format, $(\mathbf{y}, \mathbf{t})$, and reported values, $\mathbf{v}$, to a probability distribution over the set of agents. In what follows, the notation $\Phi(\mathbf{y}, \mathbf{t}, \mathbf{v})(i) = p_i$ refers to the situation when the regulator assigns probability p_i to bidder i being favored after observing the auction format $(\mathbf{y}, \mathbf{t})$ and reported values, $\mathbf{v}$. We define detection of favoritism in the following way.

Definition 5 (Detectable favoritism)**.** *Favoritism is almost surely detectable if* $\Phi(\mathbf{y}^*, \mathbf{t}^*, \mathbf{v})(1) = 1$ *for any* $v \in V$*.*

In words, favoritism can be detected if the regulator can, without uncertainty, identify the favored bidder after observing the auction format and the bids of all participants. By employing this definition, we gain further insights into the distinction between situations in which the designer cannot transfer revenue to the favored bidder (as in Proposition 1) and situations where such a transfer is possible (as in Proposition 3). In instances where positive transfers are permitted, even if the designer employs a symmetric auction mechanism, they transfer revenue to the favored bidder almost surely. Consequently, after analyzing the auction rules and bids, the regulator can establish the identity of the favored bidder.
Notably, the construction presented in Proposition 3 does not depend on the specific distribution of bidder's values, F. Hence, $M(v^*)$ also does not depend on F. Taking into account that $M(\cdot)$ is a bijection in this case, the designer can find the favored bidder's value from the auction rule as $v^* = M^{-1}(\mathbf{y}, \mathbf{t})$. With probability one, only the favored bidder bids exactly v^*. Hence, the regulator can determine the favored bidder's identity almost surely as the identity of a bidder bidding v^*. This contrasts sharply with cases in which revenue transfers are prohibited. Without knowing the distributions F, the regulator cannot determine M^{-1} and, subsequently, can not ascertain the favored bidder's value or identity based solely on the bid. This logical progression yields the following outcome as a corollary of Propositions 1 and 3.

Corollary 4 (Detection of favoritism)**.**

1. *Favoritism is not detectable in the symmetric and strategy-proof preferable auction plan* $M^*(v^*)$ *that does not allow for positive transfers.*

2. *Favoritism is detectable in the symmetric and strategy-proof preferable auction plan* $M^*(v^*)$ *that allows for positive transfers.*

This illustrates the intricate nature of detecting favoritism in practical scenarios when the designer employs the auction as a means of biased favoritism. Our findings indicate that the designer's mere

[17] If the regulator knows distribution F, it may sometimes be possible to deduce the favored bidder's identity by identifying a bid that aligns optimally with the proposed auction format as per Proposition 1. For example, if F is concave, the regulator can infer v^* from the chosen auction if v^* is sufficiently high.

utilization of a non-discriminatory auction does not guarantee the absence of favoritism; instead, it may simply render it undetectable.

5.3 Favoritism and imperfect information

Remaining with the model's interpretation of representing preferential treatment in auctions, we discuss some alternative assumptions on the designer's information concerning the favored bidder's value. Despite our contention that perfect information of the principal about the favored bidder's value is readily justifiable in some situations (since incentives are perfectly aligned), we consider an alternative assumption regarding the information structure.

Suppose that, instead of perfect knowledge about the favored bidder's value v^*, the designer's information is sufficiently precise but not perfect, i.e., for each value v^*, the designer does not know v^* but knows $v^1, v^2 \in V$ and $\delta > 0$ such that $|v^1 - v^2| < \delta$, and $v^* \in [v^1, v^2]$.

The auction described in Proposition 1 depends on perfect knowledge of v^*. We now allow the designer to pick some value $v_d^* \in [v^1, v^2]$ and employ the auction described in Proposition 1 using v_d^* rather than v^*. Evidently, such an auction is not generally the optimal auction for the favored bidder unless $v_d^* = v^*$. However, from Proposition 2, it can be inferred that such an auction will not deviate too far from the optimal one if δ is sufficiently small. This follows from the observation that any change in v^* yields only a localized impact on the pooling region that arises in optimum. Consequently, the designer can make the favored bidder better off by choosing a specific auction from the class of non-discriminatory auctions.

Regarding an auction structure that permits positive transfers, as detailed in Proposition 3, it may be worth mentioning that related results are not an artifact of perfect knowledge but feature some "continuity" on the dimension of information.

Definition 6 (Quasi-perfect favoritism). *An auction plan M implements quasi-perfect favoritism if, for any v^*, the chosen auction $M(v^*)$ provides*

$$\lim_{\delta \to 0} \mathbb{P}\left[U_1(v^*) \geq v^*\right] = 1.$$

Similarly to Section 4, quasi-perfect favoritism is implementable if there exists an auction plan, M, that implements it. This is a natural extension of perfect favoritism to a setting with imprecise information. Then, similar to Proposition 3 and Corollary 3, we can prove the following result.

Proposition 6 (Quasi-perfect favoritism). *If the designer is imperfectly informed, i.e., the designer knows $v^1, v^2 \in V$ and $\delta > 0$ such that $|v^1 - v^2| < \delta$ with $v^* \in [v^1, v^2]$, quasi-perfect favoritism is implementable even if the designer has to use symmetric and strategy-proof auctions.*

The idea behind the proof is similar to that of Proposition 3. The designer can construct an auction that transfers all collected revenue to the bidder making a bid from the interval $[v^1, v^2]$. If δ goes to zero, $[v^1, v^2]$ shrinks, and the favored bidder will be the only bidder making such a bid. Thus, if δ is sufficiently small, that is, the designer's information is sufficiently precise, the probability of revenue transfer to the favored bidder can be arbitrarily close to 1.

6 Concluding remarks

We analyze the problem of selecting an auction format that maximizes the utility of a specific bidder. We demonstrate that the appropriate choice of an auction format can favor a bidder by leveraging information about their value, even if direct discrimination is prohibited. We characterize an optimal auction for a particular bidder, for each possible value using a novel technique of majorization introduced by Kleiner et al. (2021). We extend this result to a model in which positive transfers towards bidders are permitted and show that an interim utility greater than their value can be guaranteed to that particular bidder almost surely.

In an illustrative application of the developed theory, we discuss how to prevent and detect situations of favoritism. Our results suggest that, when delegating the decision about the auction format choice to a designer, the principal should care about how much freedom should be given to the designer and how this freedom can be limited. If the goal of the principal is revenue maximization then, along with symmetry, restrictions to non-positive transfers and deterministic auctions can be imposed. A restriction on transfers helps to prevent discrimination of bidders via monetary payments. Restrictions to non-stochastic allocation rules ensure intensive competition since, otherwise, a designer can reduce competition by using lotteries to help the favored bidder.

7 Appendix

Proof of Proposition 2.

1. The existence of $\hat{v}$ follows directly from Proposition 1. For any $v^* < \bar{v}$, there exists an interval in which $\Psi_{v^*}(s) = v^*$, and thus $\mathrm{conv}\langle\Psi_{v^*}\rangle$ is affine on some interval $[\underline{x}, \overline{x})$ which implies that $\hat{v} = \underline{x}$.

2. To show monotonicity of $\hat{v}$ as a function of v^*, suppose that it is not true, i.e., there exist v_1^* and v_2^* such that $v_1^* < v_2^*$ and $\hat{v}(v_1^*) > \hat{v}(v_2^*)$. By definition of G_{v^*}, the following holds: $G_{v_2^*}(\hat{v}(v_1^*)) \geq F(\hat{v}(v_1^*))$. Since $G_{v_1^*}(v_1^*) - G_{v_1^*}(\hat{v}(v_1^*)) = 1 - F(\hat{v}(v_1^*)) > G_{v_2^*}(v_1^*) - G_{v_2^*}(\hat{v}(v_1^*))$, we must have $g_{v_1^*}(\hat{v}(v_1^*)) > g_{v_2^*}(\hat{v}(v_1^*))$. Then,

$$\begin{aligned} G_{v_1^*}(\hat{v}(v_2^*)) &< G_{v_1^*}(\hat{v}(v_1^*)) - (\hat{v}(v_1^*) - \hat{v}(v_2^*))g_{v_1^*}(\hat{v}(v_1^*)) \\ &= F(\hat{v}(v_1^*)) - (\hat{v}(v_1^*) - \hat{v}(v_2^*))g_{v_1^*}(\hat{v}(v_1^*)) \\ &< F(\hat{v}(v_1^*)) - (\hat{v}(v_1^*) - \hat{v}(v_2^*))g_{v_2^*}(\hat{v}(v_1^*)) = F(\hat{v}(v_2^*)) \end{aligned}$$

   However, $G_{v_1^*}(\hat{v}(v_2^*)) < F(\hat{v}(v_2^*))$ is impossible by construction of $G_{v_1^*}$. Thus, $\hat{v}(v^*)$ has to be monotone.

3. If v^* changes, the change of $M^*(v^*)$ is related to the change of the function $G_{v^*}(v)$. The only change of this function happens on the subset $[\hat{v}(v^*), \bar{v}]$, which is a pooling region. Since $\hat{v}(v^*)$ is increasing in v^*, the size of the pooling region is larger when v^* is smaller.

4. Define $s := \sup_{v \in V} F(v)/v$. Since $f(0) < \infty$, such a supremum exists. Then, define $v_l := 1/s$. We want to establish that $M^*(v^*)$ is a simple lottery for any $v^* \leq v_l$. Indeed, take some $v^* \leq v_l$. Then,
$$\frac{1}{v^*} \geq \frac{1}{v_l} = s.$$
This means that $G_{v^*}(z) = z/v^*$. Then, from Proposition 1, it follows that $M(v^*)$ is a simple lottery.

□

Proposition of Corollary 1.

1. If $F(\cdot)$ is convex or linear, from part 4 in Proposition 2 it follows that $s = 1/\bar{v}$. Thus, $v_l = \bar{v}$. Hence, a simple lottery is optimal for any $v^* \in V$.

2. If $F(\cdot)$ is concave, it follows from Proposition 1, that $M(v^*)$ has only one pooling region at the top, in which a lottery is used to determine a winner. The size of this pooling region is given by $\hat{v}(v^*)$.

□

Proof of Proposition 3.

Step 1 (construction of $h'(\cdot)$): Consider some symmetric and strategy-proof auction $(\mathbf{y}, \mathbf{t})$ that has the allocation rule $\mathbf{y}(\mathbf{v})$ and the transfer rule $\mathbf{t}(\mathbf{v})$. The new constructed auction, $(\mathbf{y}, \mathbf{t}')$, also has to be strategy-proof. By (5) and (4), functions $h(\cdot)$, $h'(\cdot)$ have to satisfy
$$\begin{aligned} t_i(\mathbf{v}) &= -v_i y_i(\mathbf{v}) + h(\mathbf{v}_{-i}) + \int_0^{v_i} y_i(\mathbf{v}|v_i = q)\mathrm{d}q \\ t'_i(\mathbf{v}) &= -v_i y_i(\mathbf{v}) + h'(\mathbf{v}_{-i}) + \int_0^{v_i} y_i(\mathbf{v}|v_i = q)\mathrm{d}q \end{aligned} \tag{14}$$
Transfers should almost surely be the same only in equilibrium. In equilibrium, bidder 1 always reports v^*. Hence, only vectors $\mathbf{v} = (v^*, \mathbf{v}_{-1})$ can be on the equilibrium path. For any $\mathbf{v}_{-i} \in V^{n-1}$, define
$$h'(\mathbf{v}_{-i}) := h(\mathbf{v}_{-i})$$
if at least one component of $\mathbf{v}_{-i}$ is equal to v^* and
$$\begin{aligned} h'(\mathbf{v}_{-i}) :=& v^* y_i(\mathbf{v}|v_i = v^*) - \int_0^{v^*} y_i(\mathbf{v}|v_i = q)\mathrm{d}q \\ &+ \sum_{j \neq i} v_j y_j(\mathbf{v}|v_i = v^*) - \sum_{j \neq i} \int_0^{v_j} y_j(\mathbf{v}|v_i = v^*, v_j = q)\mathrm{d}q - \sum_{j \neq i} h'(\mathbf{v}_{-j}|v_i = v^*) \end{aligned} \tag{15}$$
if none of components of $\mathbf{v}_{-i}$ is equal to v^*, in which $h'(\mathbf{v}_{-j}|v_i = v^*)$ means that the value of component v_i in $\mathbf{v}_{-j}$ is evaluated at v^*.

Step 2 (computing transfers): Equation (14) then uniquely defines $t'_i(\mathbf{v})$ given $y_i(\mathbf{v})$ and $h'(\mathbf{v}_{-i})$. Thus, if $\mathbf{v}_{-i}$ has a component equal to v^*, then $h'(\mathbf{v}_{-i}) = h(\mathbf{v}_{-i})$ and, hence,

$$t'_i(\mathbf{v}) \ = t_i(\mathbf{v}). \tag{16}$$

If all components of $\mathbf{v}_{-i}$ are different from v^*, plugging the expression (15) to (14), using $h'(\mathbf{v}_{-j}|v_i = v^*) = h(\mathbf{v}_{-j}|v_i = v^*)$, $j \neq i$ we obtain

$$\begin{aligned} t'_i(\mathbf{v}) =& - v_i y_i(\mathbf{v}) + \int_{v^*}^{v_i} y_i(\mathbf{v}|v_i = q)\mathrm{d}q + v^* y_i(\mathbf{v}|v_i = v^*) + \sum_{j\neq i} v_j y_j(\mathbf{v}|v_i = v^*) \\ & - \sum_{j\neq i} \int_0^{v_j} y_j(\mathbf{v}|v_i = v^*, v_j = q)\mathrm{d}q - \sum_{j\neq i} h(\mathbf{v}_{-j}|v_i = v^*) \\ =& - v_i y_i(\mathbf{v}) + \int_{v^*}^{v_i} y_i(\mathbf{v}|v_i = q)\mathrm{d}q + v^* y_i(\mathbf{v}|v_i = v^*) + \sum_{j\neq i} v_j y_j(\mathbf{v}|v_i = v^*) \\ & - \sum_{j\neq i} t_j(\mathbf{v}|v_i = v^*) - \sum_{j\neq i} v_j y_j(\mathbf{v}|v_i = v^*) \\ =& - v_i y_i(\mathbf{v}) + \int_{v^*}^{v_i} y_i(\mathbf{v}|v_i = q)\mathrm{d}q + v^* y_i(\mathbf{v}|v_i = v^*) - \sum_{j\neq i} t_j(\mathbf{v}|v_i = v^*), \end{aligned} \tag{17}$$

in which we also used the envelope condition (4), which ensures that

$$\sum_{j\neq i} \int_0^{v_j} y_j(\mathbf{v}|v_i = v^*, v_j = q)\mathrm{d}q + \sum_{j\neq i} h(\mathbf{v}_{-j}|v_i = v^*) = \sum_{j\neq i} t_j(\mathbf{v}|v_i = v^*) + \sum_{j\neq i} v_j y_j(\mathbf{v}|v_i = v^*)$$

Now, we need to verify that the constructed auction satisfies symmetry and it implements the described transfers in equilibrium almost surely.

Step 3 (check symmetry of $(\mathbf{y}, \mathbf{t}')$): The allocation rule in auction $(\mathbf{y}, \mathbf{t}')$ is symmetric. Now, consider $\mathbf{t}'(\mathbf{v})$. If $\mathbf{v}_{-i}$ has a component equal to v^*, then $t'_i(\mathbf{v}) = t_i(\mathbf{v})$. Since $t_i(\mathbf{v})$ is symmetric, then $t'_i(\mathbf{v})$ is also symmetric. If all components of $\mathbf{v}_{-i}$ are different from v^*, then $t'_i(\mathbf{v})$ is described by expression (17). Since all terms in (17) are symmetric, symmetry (6) is satisfied.

Step 4 (equilibrium transfers): In equilibrium, bidder 1 reports v^*. Hence, (16) implies that $t'_i(v^*, \mathbf{v}_{-1}) = t_i(v^*, \mathbf{v}_{-1})$ for all bidders, except bidder 1. Since the number of bidders is finite and the distributions are strictly increasing, the probability that some other bidder is going to report v^* is zero. Thus, the bidder 1's transfer in equilibrium is described by (17) almost surely, by plugging $v_1 = v^*$, we obtain

$$t'_1(v^*, \mathbf{v}_{-1}) = -\sum_{j\neq 1} t_j(v^*, \mathbf{v}_{-1}) = t_1(v^*, \mathbf{v}_{-1}) + R(v^*, \mathbf{v}_{-1}).$$

The no-deficit requirement is trivially satisfied in equilibrium because the constructed auction transfers all revenue to bidder 1 making the budget balanced only if bidder 1 reports v^*. In the zero probability event that some other bidder has value v^*, the constructed auction implements the same transfers as the original auction, i.e., $t'_i = t_i$ for any bidder i. This completes the proof.

□

Proof of Proposition 5.

Chen and Knyazev (2023) show that, the utility of any bidder in a second-price auction with a flexible reserve price must be $U_i(\mathbf{v}) = \int_0^{v_i} y_i(\mathbf{v}|v_i = q)dq$, in which $y_i(\mathbf{v}) = 1$ if $v_i > \max_{j\neq i}(v_j, r_i(\mathbf{v}_{-i}))$ and $y_i(\mathbf{v}) = 0$ if $v_i < \max_{j\neq i}(v_j, r_i(\mathbf{v}_{-i}))$. The choice of a reserve value function completely determines the auction format. Hence, the utility of each bidder, including bidder 1, can be written as follows

$$\begin{aligned}U_i(\mathbf{v}) &= \int_{\max_{j\neq i}(v_j, r_i(\mathbf{v}_{-i}))}^{v_i} y_i(\mathbf{v}|v_i = q)\mathrm{d}q \\ &= \max\{0, v_i - \max_{j\neq i}(v_j, r_i(\mathbf{v}_{-i}))\}\end{aligned}$$

Hence, making positive reserve prices can only reduce the utility of each bidder, including bidder 1. Thus, it is optimal to impose a zero reserve price, so $r_i(\mathbf{v}_{-i}) = 0 \ \forall \mathbf{v}_{-i} \in \mathbb{R}^{n-1}$. □

Proof of Proposition 6.

The proof is done by construction and employs auctions that satisfy the following two properties.

Definition 7 (regular auction). *A direct auction* $(\mathbf{y}, \mathbf{t})$ *is called regular if the following two conditions are satisfied*

1. *The ex-post revenue* $R(\mathbf{v})$ *is increasing in* v_i *for any* i.

2. $\mathbf{y}(\cdot)$ *and* $\mathbf{t}(\cdot)$ *are continuous, for any* i, *at each* v_i *such that* $\nexists j \neq i : v_i = v_j$.

Intuitively, an auction is regular if the revenue function is increasing with bids and the rules are continuous functions at every point where there are no equal bids. All standard formats, e.g., a first-price auction, a second-price auction and an all-pay auction satisfy it. This proof is a modified version of the proof of Proposition 5. For any regular symmetric and strategy-proof auction $(\mathbf{y}, \mathbf{t})$ that generates ex-post equilibrium revenue $R(v^*, \mathbf{v}_{-1}) = -\sum_{i=1}^{n} t_i(v^*, \mathbf{v}_{-1})$ given that bidder 1's value is v^*, we construct another direct feasible symmetric and strategy-proof auction $(\mathbf{y}, \mathbf{t}')$ in the following way.

Step 1 (construction of $h'(\cdot)$): Since $(\mathbf{y}, \mathbf{t})$ and $(\mathbf{y}, \mathbf{t}')$ are strategy-proof and symmetric, (5) implies that functions $h(\cdot)$, $h'(\cdot)$ have to satisfy

$$\begin{aligned}t_i(\mathbf{v}) &= -v_i y_i(\mathbf{v}) + h(\mathbf{v}_{-i}) + \int_0^{v_i} y_i(\mathbf{v}|v_i = q)\mathrm{d}q \\ t'_i(\mathbf{v}) &= -v_i y_i(\mathbf{v}) + h'(\mathbf{v}_{-i}) + \int_0^{v_i} y_i(\mathbf{v}|v_i = q)\mathrm{d}q \qquad (18)\end{aligned}$$

For any $\mathbf{v}_{-i} \in V^{n-1}$, define
$$h'(\mathbf{v}_{-i}) := h(\mathbf{v}_{-i})$$

if at least one component of $\mathbf{v}_{-i}$ belongs to $[v^1, v^2]$ and

$$\begin{aligned} h'(\mathbf{v}_{-i}) := & v^1 y_i(\mathbf{v}|v_i = v^1) - \int_0^{v^1} y_i(\mathbf{v}|v_i = q)\mathrm{d}q \\ & + \sum_{j\neq i} v_j y_j(\mathbf{v}|v_i = v^1) - \sum_{j\neq i}\int_0^{v_j} y_j(\mathbf{v}|v_i = v^1, v_j = q)dq - \sum_{j\neq i} h'(\mathbf{v}_{-j}|v_i = v^1) \end{aligned} \tag{19}$$

if none of components of $\mathbf{v}_{-i}$ belongs to $[v^1, v^2]$, in which $h'(\mathbf{v}_{-j}|v_i = v^1)$ means that the value of component v_i in $\mathbf{v}_{-j}$ is replaced by v^1.

Step 2 (computing transfers): Equation (18) then uniquely defines $t'_i(\mathbf{v})$ given $y'_i(\mathbf{v})$ and $h'(\mathbf{v}_{-i})$. Thus, if $\mathbf{v}_{-i}$ has a component that belongs to $[v^1, v^2]$, then $h'(\mathbf{v}_{-i}) = h(\mathbf{v}_{-i})$ and, hence,

$$t'_i(\mathbf{v}) = t_i(\mathbf{v}).$$

If all components of $\mathbf{v}_{-i}$ are different from v^*, plugging the expression (19) to (18), using $h'(\mathbf{v}_{-j}|v_i = v^1) = h(\mathbf{v}_{-j}|v_i = v^1)$, $j \neq i$ we obtain

$$\begin{aligned} t'_i(\mathbf{v}) = & - v_i y_i(\mathbf{v}) + \int_{v^1}^{v_i} y_i(\mathbf{v}|v_i = q)\mathrm{d}q + v^1 y_i(\mathbf{v}|v_i = v^1) + \sum_{j\neq i} v_j y_j(\mathbf{v}|v_i = v^1) \\ & - \sum_{j\neq i}\int_0^{v_j} y_j(\mathbf{v}|v_i = v^1, v_j = q)\mathrm{d}q - \sum_{j\neq i} h(\mathbf{v}_{-j}|v_i = v^1) \\ = & - v_i y_i(\mathbf{v}) + \int_{v^1}^{v_i} y_i(\mathbf{v}|v_i = q)\mathrm{d}q + v^1 y_i(\mathbf{v}|v_i = v^1) + \sum_{j\neq i} v_j y_j(\mathbf{v}|v_i = v^1) \\ & - \sum_{j\neq i} t_j(\mathbf{v}|v_i = v^1) - \sum_{j\neq i} v_j y_j(\mathbf{v}|v_i = v^1) \\ = & - v_i y_i(\mathbf{v}) + \int_{v^1}^{v_i} y_i(\mathbf{v}|v_i = q)\mathrm{d}q + v^1 y_i(\mathbf{v}|v_i = v^1) - \sum_{j\neq i} t_j(\mathbf{v}|v_i = v^1) \end{aligned} \tag{20}$$

in which we also used that

$$\sum_{j\neq i}\int_0^{v_j} y_j(\mathbf{v}|v_i = v^1, v_j = q)\mathrm{d}q + \sum_{j\neq i} h(\mathbf{v}_{-j}|v_i = v^1) = \sum_{j\neq i} t_j(\mathbf{v}|v_i = v^1) + \sum_{j\neq i} v_j y_j(\mathbf{v}|v_i = v^1).$$

Now, we need to verify that the constructed auction satisfies symmetry and it implements the described transfers with the probability converging to 1 when $\delta \to 0$ in equilibrium.

Step 3 (check symmetry of $(\mathbf{y}, \mathbf{t}')$):

Consider $\mathbf{t}'(\mathbf{v})$. If $\mathbf{v}_{-i}$ has a component that belongs to $[v^1, v^2]$, then $t'_i(\mathbf{v}) = t_i(\mathbf{v})$. Since $t_i(\mathbf{v})$ is symmetric, then $t'_i(\mathbf{v})$ is also symmetric. If all components of $\mathbf{v}_{-i}$ do not belong to $[v^1, v^2]$, then $t'_i(\mathbf{v})$ is described by expression (20), which has only symmetric functions inside. Thus, symmetry is satisfied.

Step 4 (equilibrium transfers):

In equilibrium, bidder 1 reports $v^* \in [v^1, v^2]$. Hence, $t'_i(v^*, \mathbf{v}_{-1}) = t_i(v^*, v_{-1})$ for each bidder $i : v_i \notin [v^1, v^2]$. If there are other bidders, except bidder 1, who have reported values in $[v^1, v^2]$, the outcomes of $(\mathbf{y}, \mathbf{t}')$ and $(\mathbf{y}, \mathbf{t})$ are the same. Since the number of bidders is finite and the distributions are strictly increasing, the probability that some other bidder i has $v_i \in [v^1, v^2]$ is $\Pr(v_i \in [v^1, v^2]) = F_i(v^2) - F_i(v^1)$. Thus, with probability $\prod_{i \neq 1} [1 - (F_i(v^2) - F_i(v^1))]$, bidder 1 is the only bidder making bid in $[v^1, v^2]$. In this case, the equilibrium transfer is the same for all bidders except for bidder 1, i.e., $t'_i(\mathbf{v}) = t_i(\mathbf{v})$ for any $i \neq 1$, and bidder 1 receives transfer described by plugging $v_1 = v^*$ to (20)

$$t'_1(v^*, \mathbf{v}_{-1}) = -\, v^* y_i(v^*, \mathbf{v}_{-1}) + \int_{v^1}^{v^*} y_i(q, \mathbf{v}_{-1}) \mathrm{d}q + v^1 y_i(v^1, \mathbf{v}_{-1}) - \sum_{j \neq i} t_j(v^1, \mathbf{v}_{-1}).$$

Thus, in this case, equilibrium revenue $R'(v^*, \mathbf{v}_{-1})$ is

$$\begin{aligned} R'(v^*, \mathbf{v}_{-1}) =& -\, t'_1(v^*, \mathbf{v}_{-1}) - \sum_{j \neq 1} t'_j(v^*, \mathbf{v}_{-1}) \\ =& v^* y_1(v^*, \mathbf{v}_{-1}) - \int_{v^1}^{v^*} y_1(q, \mathbf{v}_{-1}) \mathrm{d}q - v^1 y_1(v^1, \mathbf{v}_{-1}) \\ &+ \sum_{j \neq 1} t_j(v^1, \mathbf{v}_{-1}) - \sum_{j \neq 1} t_j(v^*, \mathbf{v}_{-1}) \\ =& U(v^*, \mathbf{v}_{-1}) + R(v^*, \mathbf{v}_{-1}) - U(v^1, \mathbf{v}_{-1}) - R(v^1, \mathbf{v}_{-1}) - \int_{v^1}^{v^*} y_1(q, \mathbf{v}_{-1}) \mathrm{d}q \\ =& R(v^*, \mathbf{v}_{-1}) - R(v^1, \mathbf{v}_{-1}), \end{aligned}$$

in which we used that

$$v_1 y_1(v_1, \mathbf{v}_{-1}) - \sum_{j \neq 1} t_j(v_1, \mathbf{v}_{-1}) = U(v_1, \mathbf{v}_{-1}) - R(v_1, \mathbf{v}_{-1}),$$

$$U(v^*, \mathbf{v}_{-1}) - U(v^1, \mathbf{v}_{-1}) - \int_{v^1}^{v^*} y_1(q, \mathbf{v}_{-1}) \mathrm{d}q = 0.$$

Since $(\mathbf{y}, \mathbf{t})$ is a regular auction and $v^1 \leq v^*$, $R(v^*, \mathbf{v}_{-1}) - R(v^1, \mathbf{v}_{-1}) \geq 0$. Thus, $R'(v^*, \mathbf{v}_{-1}) \geq 0$ and, hence, $(\mathbf{y}, \mathbf{t}')$ is also a feasible auction.

Step 5 (convergence): Since, for each bidder i, F_i is a continuous distribution function,

$$\lim_{\delta \to 0} \prod_{i \neq 1} [F_i(v^2) - F_i(v^1)] = 0.$$

Thus, in the limit as $\delta \to 0$, bidder 1 is, almost surely, the only bidder bidding $v^* \in [v^1, v^2]$. In this case,

$$
\begin{aligned}
\lim_{\delta\to 0} t_1'(\mathbf{v}) &= \lim_{\delta\to 0}(-v^* y_i(v^*, \mathbf{v}_{-1}) + \int_{v^1}^{v^*} y_i(q, \mathbf{v}_{-1})\mathrm{d}q + v^1 y_i(v^1, \mathbf{v}_{-1}) - \sum_{j\neq i} t_j(v^1, \mathbf{v}_{-1}) \\
&= \lim_{\delta\to 0}(-\sum_{j\neq i} t_j(v^1, \mathbf{v}_{-1})) = -\sum_{j\neq 1} t_j(v^*, \mathbf{v}_{-1}) = t_1(v^*, \mathbf{v}_{-1}) + R(v^*, \mathbf{v}_{-1})
\end{aligned}
$$

in which we used that, for a regular auction,

$$
\lim_{\delta\ \to 0} y_i(v^1, \mathbf{v}_{-1}) = y_i(v^*, \mathbf{v}_{-1}) \text{ and } \lim_{\delta\to 0} t_j(v^1, \mathbf{v}_{-1}) = y_j(v^*, \mathbf{v}_{-1}).
$$

To conclude the proof, notice that the second-price auction is a regular, symmetric, and strategy-proof auction. Hence, similar to Proposition 5, this implies the possibility of quasi-perfect favoritism.

□

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